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Residual strength of composite laminates with a centre crack under tension

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Abstract—An empirical relationship between the failure stress and stress intensity factor at failure is examined for the estimation of residual strength of various types of laminates with different layup sequence and having centre cracks. The estimated fracture strengths for these laminates under tension are compared with existing experimental results. They are found to be in reasonably good agreement with each other.

Keywords: Residual strength; composite laminates; cracks; failure assessment diagram.

1. INTRODUCTION

The adoption of advanced composites by the aerospace industry triggered a massive research effort. Obviously, the ability to design a structure composed of fibre reinforced plastics must be matched by the ability to fabricate the component as well as by the knowledge of damage, failure, and fracture behaviour in its service time. The fracture phenomena of fibrous composite materials are complex in nature, owing to the complexity and diversity of composite systems and lay-up constructions. Unlike metallic materials, a composite (or a composite laminate) containing a hole or a notch fractures in a complicated way.

The problem of whether linear elastic fracture mechanics (LEFM) established for metal materials is applicable to fibre composites has long been investigated by many researchers. It is noted that when the crack surface is smooth and straight and the crack propagates with self-similarity, the fracture problem can be analysed by LFEM; otherwise, the results of LEFM may not be correct. Because in most cases the crack surfaces are neither smooth nor self-similar, the theories and criteria of LEFM must be modified to be suitable for a composite structure.

In the course of developing fracture mechanics for composite materials, many theories and models have been proposed. One of the best recognised applications of LEFM to composites on the macroscopic scale is that of Waddoups *et al*. [1], the WEK-fracture models, known as the inherent flaw models. Whitney and Nuismer [2, 3] suggested two stress fracture criteria known as the point stress criterion (PSC) and the average stress criterion (ASC). Both stress fracture criteria are two-parameter models based on the unnotched strength and a characteristic dimension which were assumed to be material constants for a given composite material system and lay-up. Pipes *et al*. [4, 5] and Kim *et al*. [6] modified the PSC and proposed a three-parameter model with an exponential relationship between the characteristic dimension and the notch size. These theories are concerned primarily with the prediction of onset of fracture.

Aronsson and Backlund [7] have proposed the damage zone model (DZM) to evaluate the residual strength of composite laminates, which requires only the basic properties of the laminates, such as the unnotched strength, apparent fracture energy and stiffness parameters. The evaluation of the damage zone model was based on the finite element method which is less attractive from a practical point of view than other closed-form criteria. Eriksson and Aronsson [8] proposed a semi-empirical two-parameter model known as the damage zone criterion (DZC), based on the equilibrium condition and the Dugdale stress distribution in the damage zone. This model simulated the damage zone by a fictitious crack in the stress intense region at the tip of the notch. The critical damage length was determined through correlation with experimental data, which was in line with a characteristic distance. In the previous approaches, the models were normally developed to find out the stress state and/or corresponding damage at a given applied load.

Khatibi *et al*. [9] have evaluated the residual strength of composite laminates with a sharp notch using an effective crack growth model (ECGM). Damage is assumed to initiate when the local normal stress ahead of the notch tip reaches the tensile strength of the unnotched laminate. The damage was modelled by a fictitious crack with cohesive stress acting on crack surfaces and the damage growth was simulated by extension of the fictitious crack and reduction of cohesive stress with crack opening. The apparent fracture energy (G_{C}) was used to define the relationship between the unnotched strength and the critical crack opening. Based on the global equilibrium condition, an iterative technique was developed to evaluate the applied load required to produce the damage growth. The residual strength of notched composite laminates was defined by the applied load instability with damage growth. The effect of damage increment on the convergence of the residual strength was investigated. They claimed that the residual strength simulated from their new model correlated well with existing experimental data for different laminate configurations. It is noted from their results that there is a large variation in G_{C} values and hence taking average of these values as basic input in their analysis will be erroneous. It is very interesting to note that the fracture stress σ_{f} decreases with the apparent fracture toughness $K_{\max}$ (or the apparent fracture energy G_{C}), which indicates a relationship between $K_{\max}$ and σ_{f} (or G_{C} and σ_{f}) as observed in metallic materials [10, 11]. Since the intensity of the stress at the crack-tip,

K, is a function of load, geometry and crack size, it is more appropriate to have a relationship between the stress intensity factor at failure ($K_{\max}$) and the failure stress (σ_{f}) from the fracture data of cracked specimens. Establishment of such a relationship will be useful for the development of failure assessment diagram and for the determination of fracture strength of any cracked configurations. A relationship which has been successfully applied to metallic structures containing cracks, is in the form [11]

$$K_{\max} = K_{\mathrm{F}}\{1 - m(\sigma_{\mathrm{f}}/\sigma_{\mathrm{u}}) - (1 - m)(\sigma_{\mathrm{f}}/\sigma_{\mathrm{u}})^{p}\}, \quad (1)$$

where $K_{\max}$ is the elatic stress intensity factor at failure, σ_{f} is the fracture strength, σ_{u} the ultimate tensile strength of the material, K_{F}, m and p are the three material fracture parameters to be determined from the test data of cracked specimens.

The objective of the present study is to compute the fracture strength of various types of centre-cracked composite laminates with different lay-up sequence using the relationship given in equation (1), and to compare with the existing test results as well as those predicted in Ref. [9] from the point stress criterion (PSC), damage zone criterion (DZC) and the effective crack growth model (ECGM).

2. DEVELOPMENT OF AN EMPIRICAL RELATIONSHIP BETWEEN $K_{\max}$ AND σ_{f}

It is well known fact that the tensile strength, σ_{f}, of a specimen decreases with increase in the size of the crack. In the absence of cracks, the fracture strength, σ_{f} of a specimen equals the ultimate tensile strength, σ_{u} of the material, and the elastic stress intensity factor at failure, $K_{\max}$ becomes zero. When the fracture strength, σ_{f} of a cracked specimen is less than the yield strength σ_{ys} of the metallic material, then there exists a linear relationship between σ_{f} and $K_{\max}$ [10]. For small crack sizes, where $\sigma_{\mathrm{ys}} < \sigma_{\mathrm{f}} < \sigma_{\mathrm{u}}$, the relationship between $K_{\max}$ and σ_{f} is expected to be non-linear. The idea of expressing $K_{\max}$ as a function of $\sigma_{\mathrm{f}}/\sigma_{\mathrm{u}}$ in equation (1), is mainly for estimation of failure strength of a cracked body, whether the crack existing in it is through-the-thickness or part-through thickness, small or large in size, from a single expression. In the absence of cracks, $\sigma_{\mathrm{f}} \to \sigma_{\mathrm{u}}$ and $K_{\max} \to 0$. In order to account this limiting condition,the expression for $K_{\max}$ given by equation (1) in terms of $\sigma_{\mathrm{f}}/\sigma_{\mathrm{u}}$, is more appropriate. The exponential form of the third term in equation (1) is preferred to represent the non-linear variation of $K_{\max}$ with σ_{f}, when $\sigma_{\mathrm{f}} > \sigma_{\mathrm{ys}}$. In order to determine the three fracture parameters (K_{F}, m and p) in equation (1), tensile strength values from three different cracked configurations are required. The accuracy in the generation of the failure assessment diagram from equation (1) will improve, if the strength value of a cracked specimen is above the yield strength, σ_{ys} material and the other two strength values of cracked specimens are below σ_{ys}.

In composite materials, two different regimes may be identified. When defects are small, general damage such as yielding in metals occurs as dispersed small cracks in the matrix, fibre breakages, or interface debonding; but when a defect is

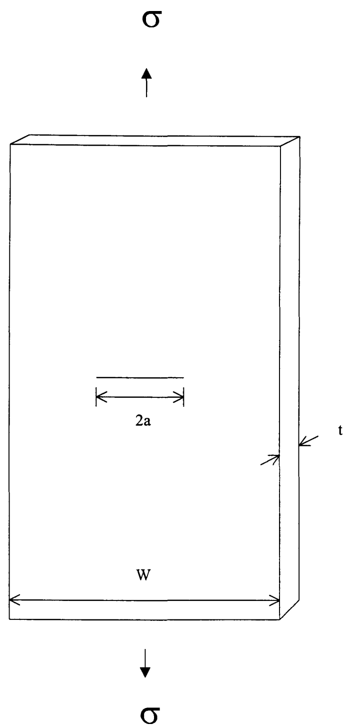

Figure 1. Centre-through crack specimen under tension.

large, it may grow as a single crack, leading to final fracture with little damage or deformation in the material remote from the crack tip. Generally, there is no clear-cut boundary dividing the two regimes. For many kinds of materials, in the earlier stage of fracture, it falls in the general damage fracture regime, while in the later stages larger damage areas (or cracks) will grow faster than others; eventually, the largest crack leads to final fracture.

The procedure for evaluating the three parameters K_F, m and p in equation (1) is described here by considering the test data of centre crack tension specimens for $[0/\pm 45/90]_{2S}$ carbon fibre/epoxy (T300/N5208) which is designated as type C laminate [9].

The stress intensity equation for a centre-through crack specimen under tension (see Fig. 1) is given by [12]

$$K = \sigma\{\pi a \sec(\pi a/w)\}^{1/2}, \tag{2}$$

where a is half the crack length, w is the specimen width, and σ is the applied stress. Using equation (2), the stress intensity factor at failure K_{max} is calculated from the experimental data, viz. fracture strength (σ_f), crack size ($2a$) and the specimen width (w). The results presented in Table 1, were found to fit the values of K_{max} and σ_f in the linear relation,

$$K_{max} = K_F\{1 - m(\sigma_f/\sigma_u)\}, \tag{3}$$

Table 1.
Fracture data for $[0/\pm45/90]_{2S}$ carbon fibre/epoxy (T300/N5208) laminate designated by type C

Width (mm)	Crack length $2a$ (mm)	Failure Stress σ_f (MPa) Test [9]	K_{max} $(\mathrm{MPa}\sqrt{\mathrm{m}})$ equation (2)	σ_f/σ_u
25.3	2.8	433.2	28.95	0.877
25.4	7.8	316.7	37.24	0.641
50.6	15.2	287	46.99	0.581
76.2	25.2	239.1	51.06	0.484

Fracture parameters: $K_F = 77.89\ \mathrm{MPa}\sqrt{\mathrm{m}}$, $m = 0.7323$, $p = 26.97$; unnotched strength: $\sigma_u = 494$ MPa.

through a least−squares curve fit; the relevant values are: $K_F = 77.89\ \mathrm{MPa}\sqrt{\mathrm{m}}$ and $m = 0.7323$. When the applied stress σ in the stress intensity equation (2) is equal to σ_f, the value of the stress intensity factor, K becomes K_{max}. Using equation (3), one can set up the following equation to determine σ_f for a centre-through crack specimen under tension,

$$\sigma_f = \sigma_u\left[m + (\sigma_u/K_F)\{\pi a \sec(\pi a/w)\}^{1/2}\right]^{-1}. \tag{4}$$

Since the material parameter, m for the fracture data is 0.7323, the fracture strength, σ_f corresponding to a negligibly small crack size ($2a \to 0$) from equation (4) is found to be $1.4\sigma_u$. Figure 2 shows the fracture strength *versus* crack length ($\sigma_f - 2a$) curve generated from equation (4). The fracture strength σ_f decreases monotonically from 692 MPa (which is higher than the ultimate strength 494 MPa of the material) to zero for the crack size increases from zero to the width of the plate. For smaller crack sizes, equation (4) yields the strength value higher than the ultimate strength which gives unconservative fracture strength estimation. For conservative estimations of fracture strength to the smaller size of cracks in structural components, Refs. [13, 14] suggest modifications in the theoretical fracture strength curve by drawing a tangent line from the value of the ultimate tensile strength (σ_u) on the fracture strength axis to the $\sigma_f - 2a$ curve and omitting the portion of the theoretical $\sigma_f - 2a$ curve above the tangent line and considering the tangent line with the remaining portion of the theoretical curve. The data generated from the tangent line of the $\sigma_f - 2a$ curve shows non-linear variation between K_{max} and σ_f, whereas the remaining portion of the $\sigma_f - 2a$ curve represents the linear relation between K_{max} and σ_f. In order to represent the variation of K_{max} with σ_f in a single equation, the generated data of K_{max} and σ_f from the tangent line of $\sigma_f - 2a$ curve is substituted in equation (1) to determine the unknown parameter, p through a least−squares curve fit. Otherwise, the unknown parameter, p in the relation (1) can be determined directly as follows.

For a wider centre-through cracked plate specimen (i.e. w is very large) having a smaller crack size, $2a$ (which implies that $2a/w \to 0$) or an infinite centre-through

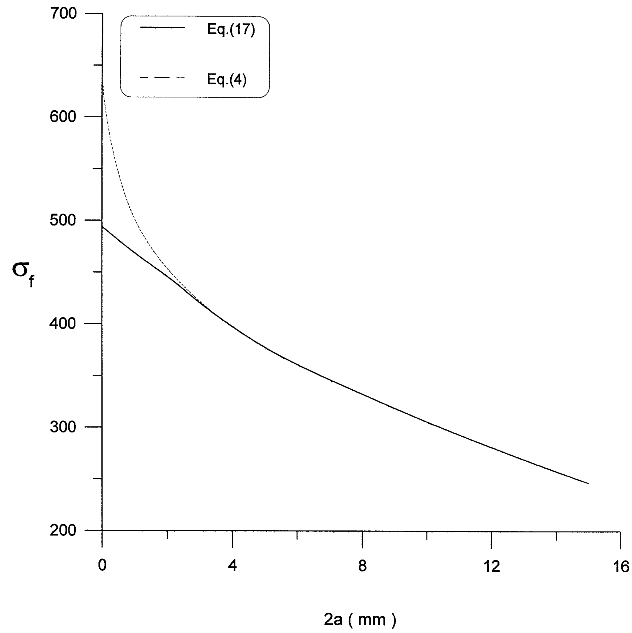

Figure 2. Fracture strength *versus* crack length curve for $[0/\pm45/90]_{2S}$ carbon/epoxy (T300/N5208) laminate (type C).

cracked plate under tension, the stress intensity equation (2) is simplified to

$$K = \sigma\sqrt{\pi a}. \tag{5}$$

Using equations (3) and (5), one can write the equation for the fracture strength curve as

$$(\sigma_f/\sigma_u) = K_F/(K_F m + \sigma_u\sqrt{\pi a}). \tag{6}$$

If the tangent line drawn from the point $(0, \sigma_u)$ to the fracture strength curve (i.e. $\sigma_f - 2a$ curve) touches at the point $(2a_0, \sigma_{f_0})$, then the equation of the tangent line is

$$\sigma_f = \sigma_u[1 - \{1 - (\sigma_{f_0}/\sigma_u)\}(a/a_0)]. \tag{7}$$

The slope of the fracture strength curve (6) at the point $(2a_0, \sigma_{f_0})$ is

$$d(\sigma_f/\sigma_u)/d(2a) = -(\sigma_{f_0}/\sigma_u)^2(\sigma_u\sqrt{\pi a_0}/4a_0 K_F). \tag{8}$$

Equating the values of K_{max} for the crack size $2a_0$ obtained from equations (3) and (5), one can get

$$\sigma_{f_0}\sqrt{\pi a_0} = K_F\{1 - m(\sigma_{f_0}/\sigma_u)\}. \tag{9}$$

Using equation (9) in equation (8), the slope of the fracture strength curve (6) at the point $(2a_0, \sigma_{f_0})$ can be written as

$$d(\sigma_f/\sigma_u)/d(2a) = -(1/4a_0)(\sigma_{f_0}/\sigma_u)\{1 - m(\sigma_{f_0}/\sigma_u)\}. \tag{10}$$

Equating the slope of the tangent line (7) with the slope of the fracture strength curve given by equation (10) at the point of contact $(2a_0, \sigma_{f_0})$, one can get the value of σ_{f_0} from the following equation

$$m(\sigma_{f_0}/\sigma_u)^2 - 3(\sigma_{f_0}/\sigma_u) + 2 = 0. \tag{11}$$

The solution of the quadratic equation (11) is

$$(\sigma_{f_0}/\sigma_u) = \{3 - (9 - 8m)^{1/2}\}/(2m). \tag{12}$$

The value of a_0 can be obtained directly from equations (9) and (12) as

$$a_0 = (1/\pi)\left[K_F\{1 - m(\sigma_{f_0}/\sigma_u)\}/(\sigma_{f_0})\right]^2. \tag{13}$$

For the crack sizes less than $2a_0$, the fracture strength corresponding to those cracks are to be obtained from equation (7). If the crack size $2a^* = a_0$, then the fracture strength σ_f^* and $K_{\max}^*$ corresponding to this crack size become

$$\sigma_f^* = (\sigma_u + \sigma_{f_0})/2, \tag{14}$$

$$K_{\max}^* = K_F K_m^*, \tag{15}$$

where $K_m^* = (1/\sqrt{2})(\sigma_f^*/\sigma_{f_0})\{1 - m(\sigma_{f_0}/\sigma_u)\}$. Using the values of σ_f^* and $K_{\max}^*$ in equation (1), one can obtain the parameter, p as

$$p = \log\left[\left\{1 - m\left(\sigma_f^*/\sigma_u\right) - K_m^*\right\}/(1 - m)\right]/\log\left(\sigma_f^*/\sigma_u\right). \tag{16}$$

It is noted from equations (12) and (14) that σ_f^*/σ_u and σ_f^*/σ_{f_0} are functions of m which imply that K_m^* is also a function of m. Thus, the material constant p in equation (16) is only a function of m. The value of m in general is greater than zero and less than unity. Whenever m is found to be greater than unity, the parameter m has to be truncated to 1.0 by suitably modifying the parameter K_F with the fracture data. If m is found to be less than zero, the parameter m has to be truncated to zero and the average of $K_{\max}$ values from the fracture data yields the parameter K_F. The third term in equation (1) becomes insignificant when m is close to unity. Figure 3 gives variation of p with m. It can be seen that p increases with m and rises asymptotically when m tends to unity. For $[0/\pm45/90]_{2S}$ carbon fibre/epoxy (T300/N5208) laminates, the third parameter, p, in equation (1) is found to be 26.97. Figure 4 shows the failure assessment diagram for the laminate (type C).

3. FRACTURE STRENGTH DETERMINATION

Using equations (1) and (2) one can set up the following equation to determine the fracture strength σ_f of a centre-through crack tensile specimen for a specified crack size.

$$(1 - m)(\sigma_f/\sigma_u)^p + [m + \{(\sigma_u\{\pi a \sec(\pi a)\}^{1/2})/K_F\}](\sigma_f/\sigma_u) - 1 = 0. \tag{17}$$

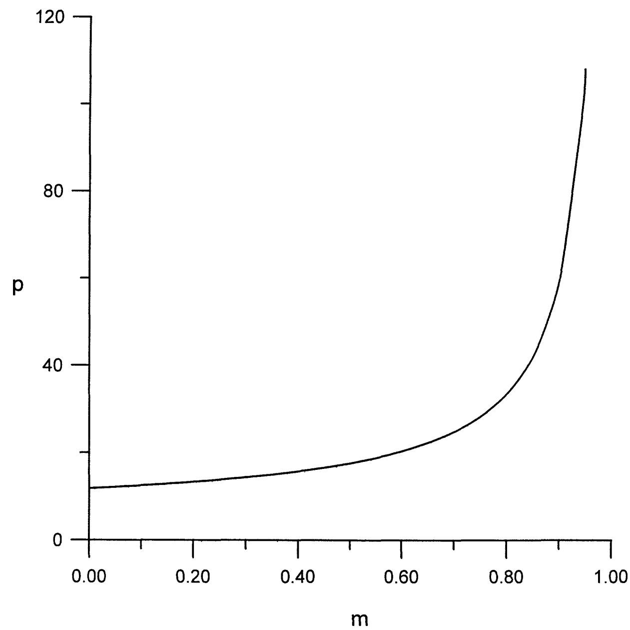

Figure 3. Relation between fracture parameters p and m.

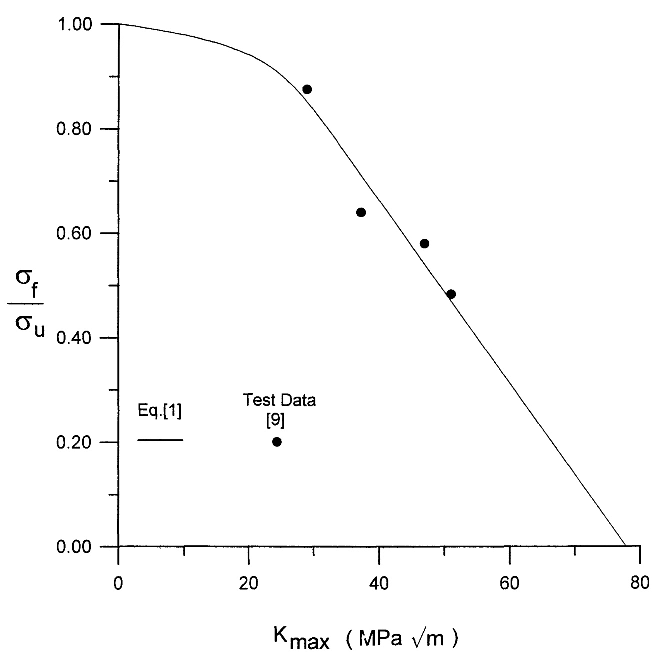

Figure 4. Failure assessment diagram for $[0/\pm 45/90]_{2S}$ carbon/epoxy (T300/N5208) laminate (type C).

The non-linear equation (17) is solved for σ_f using the Newton–Raphson iterative scheme. It can be seen from the results presented in Fig. 2 that the solutions obtained from equation (17) for smaller crack sizes are found to be less than the ultimate strength value (494 MPa), whereas equation (4) gives the strength values higher than 494 MPa. However, for large crack sizes, both the equations (4) and (17) yield the same residual strength. For different cracked configurations the corresponding stress intensity equation should be used in equation (1) to get the required fracture strength equation for residual strength evaluation.

4. RESULTS AND DISCUSSION

In order to examine the applicability of the three parameter fracture criterion for the relationship between stress intensity factor at failure (K_{max}) and the fracture strength (σ_f) as given by equation (1), five groups of experimental data [9] for residual strength of composite laminates having centre cracks are correlated with present theory. Table 2 gives the fracture parameters, viz. K_F, m and p in equation (1), generated from the fracture data for different composite laminates: carbon fibre/epoxy (T 300/914C), carbon fibre/epoxy (T300/N 5208), Glass fibre/epoxy (Scotch ply1002). Table 2 also contains the fracture parameters for 7075-T651 aluminium alloy obtained from the fracture data [15] of centre-cracked tension plates (width, $W \cong 203$ mm, thickness, $t = 8$ mm, and crack sizes ranging from 18 mm to 60 mm). Table 3 gives the fracture analysis results for the different types of composite laminates along with test results. The relative error between

Table 2.
Fracture parameters (K_F, m and p) in equation (1) for different materials

Laminate type	Material	Lay-up details	Ultimate stress σ_u (MPa)	Fracture parameters		
				K_F (MPa$\sqrt{m}$)	m	p
A	Carbon fibre/Epoxy (T300/914C)	$[(\pm 45/0/90)_3/0/90/\pm 45]_S$	548	44.80	0.1923	13.26
B	Carbon fibre/Epoxy (T300/N5208)	$[0/90]_{4S}$	636	99.07	0.9212	72.14
C	Carbon fibre/Epoxy (T300/N5208)	$[0/\pm 45/90]_{2S}$	494	77.89	0.7323	26.97
D	Glass fibre/Epoxy (Scotch Ply 1002)	$[0/\pm 45/90]_{2S}$	320	30.98	0.6807	23.79
E	Glass fibre/Epoxy (Scotch Ply 1002)	$[0/90]_{4S}$	422	33.70	0.6851	24.02
	Aluminium alloy (7075-T651)	—	594	78.34	0.5412	18.65

Table 3.
Fracture analysis for different composite laminates with a centre crack under tension

Laminate type	Width (mm)	Crack length $2a$ (mm)	Failure stress σ_f (MPa) Test [9]	Present analysis	
				Failure stress σ_f (MPa) Theory equation (17)	Relative error % equation (18)
A	36.2	6	355.7	388.1	−9.1
	48.1	8	334.3	344.6	−3.1
	60.2	10	350.7	312.6	10.9
	36.3	10	304.7	304.1	0.2
B	25.1	2.8	471.9	471	0.2
	25.1	7.8	321.2	379.2	−18.1
	50.5	15.2	322.5	322.4	0
	76	25.2	319.3	277.4	13.1
C	25.3	2.8	433.2	425.3	1.8
	25.4	7.8	316.7	334.2	−5.5
	50.6	15.2	287	279	2.8
	76.2	25.2	239.1	236.8	1.0
D	25	2.9	226.2	231.2	−2.2
	25.2	7.7	173.8	169.6	2.4
	50.6	14.7	140.8	136.9	2.8
	76	24.8	108.2	111.7	−3.2
E	25.2	2.6	272.6	283.1	−3.9
	25	7.2	203.8	202	0.9
	50.6	14.9	170.1	155.7	8.5
	76	25.8	116.1	124	−6.8

theory and test results is defined by

$$\text{relative error}(\%) = 100 \times \{1 - (\text{Theoretical result}/\text{Test result})\}. \tag{18}$$

Theoretical and experimental results are found to be in good experiment with each other. Figure 5 shows the failure assessment diagram for 7075-T651 aluminium alloy. Most of the data of Ref. [15] falls close to the failure boundary. Figure 6 shows the failure assessment diagram for glass/epoxy laminates (types D & E) along with the test data presented in Ref. [9]. It can be seen from the results presented in Figs 7 and 8 that, compared to the other fracture models (viz. PSC, DZC, and ECGM), the present simple analytical model correlates well with the experimental residual strength of composite laminates.

5. CONCLUDING REMARKS

This paper provides an empirical approach towards predicting residual strength of materials containing a central crack subjected to tensile loading. The empirical relationship between $K_{\max}$ and σ_f through fracture toughness parameters K_F, m and p

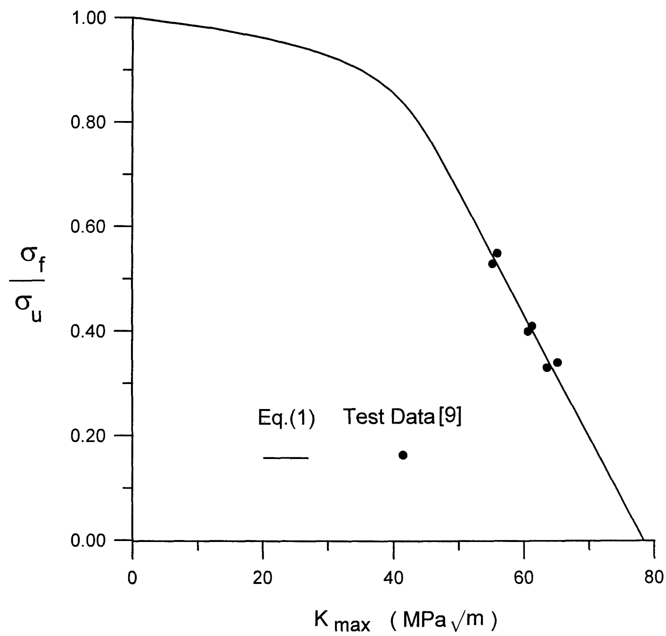

Figure 5. Failure assessment diagram for a 7075-T651 aluminium alloy.

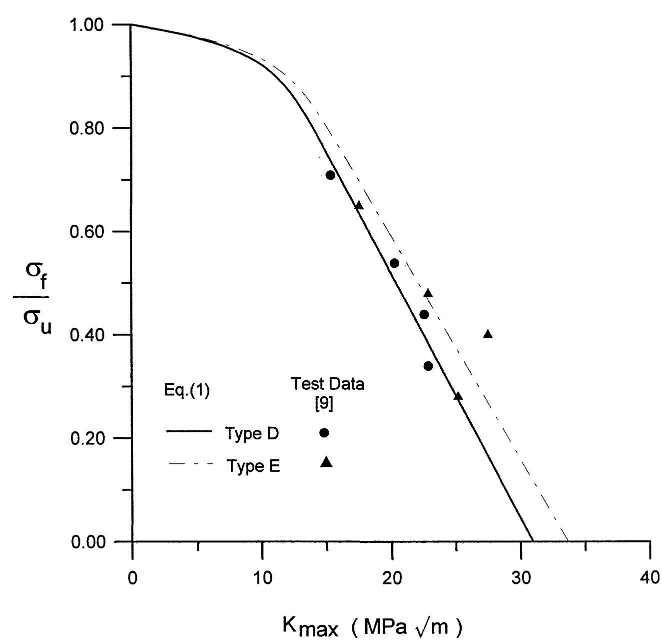

Figure 6. Failure assessment diagram for glass/epoxy laminates (types D & E).

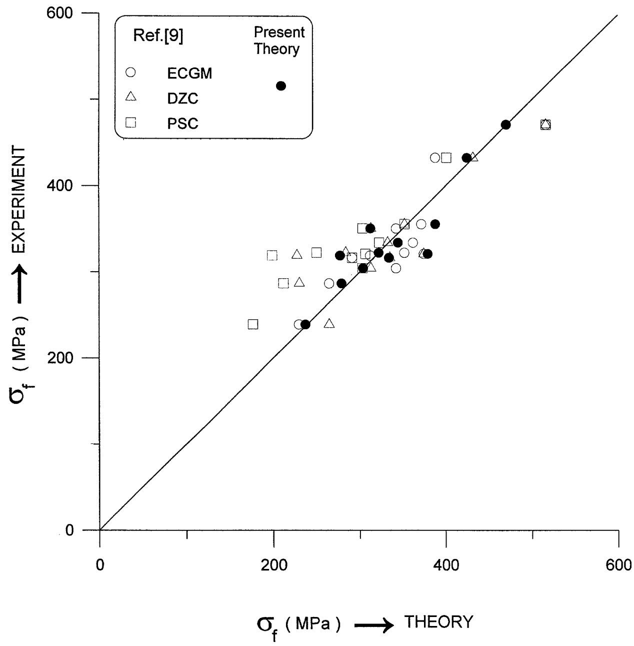

Figure 7. Comparison of experimental and theoretical residual strength, σ_f (MPa) values of carbon/epoxy laminates.

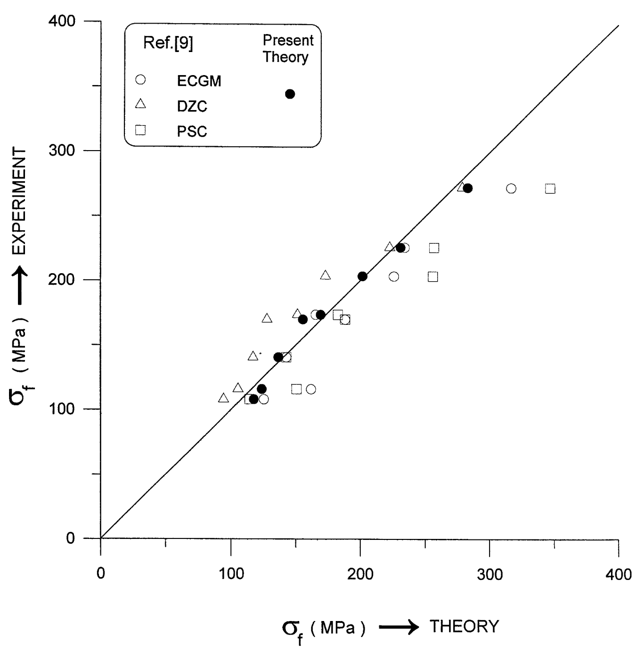

Figure 8. Comparison of experimental and theoretical residual strength, σ_f (MPa) values of glass/epoxy laminates.

in equation (1) will be useful for the determination of fracture strength of any structural components containing cracks. For the determination of the fracture toughness parameters, the test results of simple laboratory specimens like centre crack specimens can be used. Since fracture toughness parameters are dependent on laminate configuration, thickness and loading, these are to be determined from the intended laboratory specimens of similar laminate configuration. For the determination of fracture strength of any structural configuration, the stress intensity factor corresponding to the geometry has to be used in equation (1) to set up the necessary fracture strength equation which has to be solved using Newton–Raphson method. If the values of applied stress and the corresponding stress intensity factor for the crack size lie below the $K_{\max} - \sigma_{\mathrm{f}}$ curve, the structure is safe. Thus the advantage of developing failure assessment diagram is helpful in estimating the factor of safety.

REFERENCES

1. M. E. Waddoups, J. R. Eisenmann and B. E. Kaminski, Macroscopic fracture mechanics of advanced composite materials, *J. Compos. Mater.*, **5**, 446–454 (1971).
2. J. M. Whitney and R. J. Nuismer, Stress fracture criteria for laminated composites containing stress concentrations, *J. Compos. Mater.*, **8**, 253–265 (1974).
3. R. J. Nuismer and J. M. Whitney, Uniaxial failure of composite laminates containing stress concentrations, in: *Fracture Mechanics of Composites, ASTM STP 593*, pp. 117–142. American Society for Testing and Materials (1975).
4. R. B. Pipes, R. C. Wetherhold and J. W. Gillespie, Jr., Macroscopic failure of fibrous composites, *Materials Science and Engineering* **45**, 247–253 (1980).
5. R. B. Pipes, R. C. Wetherhold and J. W. Gillespie, Jr., Notched strength of composite materials, *J. Compos. Mater.*, **12**, 148–160 (1979).
6. J. K. Kim, D. S. Kim and N. Takeda, Notched strength and fracture criterion in fabric composite plates containing a circular hole, *J. Compos. Mater.*, **29**, 982–998 (1995).
7. C. G. Aronsson and J. Jan Backlund, Tensile fracture of laminates with cracks, *J. Compos. Mater.*, **20**, 287–307 (1986).
8. I. Eriksson and C. G. Aronsson, Strength of tensile loaded graphite/epoxy laminates containing cracks, open and filled holes, *J. Compos. Mater.*, **24**, 456–482 (1990).
9. A. Khatibi, L. Ye and Y. W. Mai, Effective crack growth and residual strength of composites laminates with a sharp notch, *J. Compos. Mater.*, **30**, 333–357 (1996).
10. J. C. Newman, Jr., Fracture analysis of various cracked configurations in sheet and plate materials, in: *ASTM STP 605*, pp. 104–123. American Society for Testing and Materials (1976).
11. B. Nageswara Rao and A. R. Acharya, Failure assessment on 34 Cr Mo4 grade steel cylindrical pressure vessels with an axial surface crack, *Int. J. of Pressure Vessels and Piping* **72**, 157–163 (1997).
12. H. Tada, P. C. Paris and G. R. Irwin, *The Stress Analysis of Cracks Hand Book*. Del Research Corporation, Missouri (1973).
13. Fracture Criteria in the Design of Pressure Vessels/Rocket Motor Casting, VSSC-TR-15-224-81, Joint Study by Vikram Sarabhai Space Centre, Trivandrum, National Aeronautical Laboratory, and Indian Institute of Science Bangalore, India (1981).
14. B. Nageswara Rao and A. R. Acharya, Fracture analysis of a surface cracked plate under tension, *Int. J. Engng Fracture Mechanics*, **32**, 551–559 (1989).
15. F. C. Allen, Effect of thickness on the fracture toughness of 7075 aluminium in the T6 and T 73 conditions, in: *ASTM STP 486*, pp. 16–38. American Society for Testing and Materials (1971).